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Enhancing Pig Feed Mixture: Optimization Through Goal Programming

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Abstract: Feed accounts for 70–75% of the total cost of pig production, so the composition of the feed mixture largely determines both the profitability of a farm and the growth performance of the animals. Conventional least-cost formulation treats the problem as a single-objective linear program and therefore ignores quality attributes of the mixture that decision makers care about in practice. This paper formulates pig feed design as a multicriteria problem in which three conflicting objectives are considered simultaneously: minimizing the cost of the mixture, maximizing its nutrient (digestibility) value, and minimizing its moisture content, the last being a proxy for shelf life and handling quality. Twelve locally available ingredients were considered for three growth phases of pigs (starter, up to 20 kg; grower, 20–50 kg; finisher, 50–100 kg), with nutritional bounds taken from Bureau of Indian Standards and NRC recommendations and ingredient compositions from the CVB feed tables. The marginal (single-objective) solutions and the resulting payoff matrix demonstrate that the objectives are strongly conflicting: relative to the minimum-cost ration for starter pigs, the nutrient-maximizing and moisture-minimizing rations are 62% and 81% more expensive, respectively. The multicriteria model is therefore converted into a lexicographic goal programming model in which the marginal optima serve as target values and positive and negative deviations from the targets are minimized under decision-maker priorities. Three priority scenarios are solved for each growth phase, and a fourth scenario introduces deviation variables on the crude protein constraint to control the protein oversupply observed in all earlier solutions. A weight-based sensitivity analysis implemented in Google Colab confirms that the model reallocates ingredient proportions predictably as priorities change while all scenarios remain feasible. The framework yields balanced, phase-specific rations and provides an interactive decision-support tool that can be extended to other livestock systems.

Keywords: goal programming; lexicographic priority; moisture constraint; multicriteria optimization; nutrient constraints; payoff table; pig feed formulation

1 Introduction

Feed mixtures are the blends of ingredients fed to animals as part of their nutrition. Well-balanced rations ensure animal growth and thereby help meet the food requirements of an ever-expanding population [1]. As population, income, and economic development increase, the demand for food products of animal origin rises proportionally; this demand can be met only by increasing the quantity and quality of animal feed [2]. Growth in quantity cannot rely solely on expanding the cultivated area or on importing feed and animal products, because both fertile agricultural land and the funds available for imports are limited. In this context, the sensible production of high-quality animal feed becomes vital for any economy. Any feed given to livestock must be both nutritious and affordable while enabling maximum weight gain [3].

In recent years' pig production has increased significantly worldwide [4]. At the same time, contaminated feeds and incorrect feeding practices have focused attention on the detection and minimization of health risks and nutritional deficiencies in pig meat. In pig production, feed expenses represent more than 70–75% of total costs [5]. Thus, the selection of feed ingredients that do not compete with human food use is essential for a sustainable and profitable feeding program. The most important requirement of feed formulation is to ensure that the feed is an appropriate mix that meets the pig's nutritional requirements and produces the maximum return for the least expenditure [5]. Formulating a feed requires an understanding of the energy, crude protein (CP), specific amino acid, and crude fibre requirements at the various growth stages [6]. Two indirect systems are commonly used to measure the energy content of pig feeds and the energy requirements of pigs: the digestible energy (DE) and metabolizable energy (ME) systems, and feed can be formulated using either [7]. Both systems, however, overestimate the energy content of feeds with high protein or fibre content and underestimate that of feeds with high starch or fat content. The net energy (NE) system removes the heat increment from the ME system, which allows a more precise measurement of the energy value of an ingredient, especially those rich in fibre and protein [8,9]. Vitamin and mineral supplementation at levels exceeding normal requirements is also essential [10], and nutrients other than energy are determined by laboratory testing of the feed ingredients. Depending on the ratios of the various ingredients in the ration, their availability in a country may vary from season to season, so it is important to identify the supplies that provide the most feasible and economical feed ingredients. Such a feed can be produced only by optimal blending of the ingredients [11]. Mathematical optimization methods allow the optimum mixture of ingredients to be determined for both economic and nutritional criteria [12], and with these tools it is possible to find a set of ingredients that satisfies the nutritional requirements of livestock [13] in a relatively rapid and cost-effective manner.

Feed formulation has developed on the basis of linear programming (LP), which has provided a mathematical means of describing the feeding problem since the early 1950s, when it was first introduced by Waugh [14]. Linear programming is, however, not an ideal way to approach ration formulation in practice (feed manufacturing) because of several limitations, primarily identified by Rehman and Romero [15] as the singularity of the objective function and the rigidity of the constraint set, respectively. Because an LP-based objective function restricts the preferences that the decision maker is able to express, Lara [16] also criticized LP. Since then, several different mathematical programming methods have been used in formulating livestock rations [17]. Goal programming was first used by Rehman and Romero [18] because it does not impose such strict conditions and allows several criteria to be considered in the decision. The restrictive conditions on the nutrients in animal feed were relaxed by Lara and Romero [19] using interactive multicriteria programming (the STEM approach). Bailleul et al. [20] minimized nitrogen excretion in pig diets by modifying the conventional least-cost formulation (LCF) method and using a multi-objective optimization method. Ghosh et al. [21] used goal programming for nutrient management to derive the optimum fertilizer mix for rice cultivation in West Bengal. Pomar et al. [22] presented a multi-objective optimization model, adapted from the traditional least-cost formulation program, to decrease the feed cost and total phosphorus content of pig feed. Plà [23] gave an overview of the mathematical models used in managing sow herds. Trienekens and Zuurbier [24] detailed the quality and safety requirements in the food industry, and Han et al. [25] investigated relationship and quality management in the Chinese pork supply chain. Finally, Niemi et al. [26] used stochastic dynamic programming to evaluate the value of precision feeding technologies for grow–finish pigs. Building on these limitations and advances, the present study proposes a multicriteria model that explicitly considers three aspects of feed design—cost (Z_1), nutrient sufficiency (Z_2), and moisture (Z_3)—in the decision-making process, as constraints and preferences alongside realistic nutritional constraints.

The study explores the more complex aspects of feed quality in addition to the traditional problem of optimizing feed to fulfil essential nutrient requirements at an affordable cost. It begins by adding to the conventional feed problem an additional "quality" goal, so that minimal feeding cost is sought while a desired level of the quality goal is achieved. Goal programming is then used to represent a number of preferences and objectives in the decision-making process. The main objectives of the study are: (a) to treat feed mixture optimization as a problem involving more than the cost of the mix; (b) to build a multicriteria model that joins three components—cost, nutrient sufficiency, and moisture; and (c) to apply goal programming to this model, since it is an effective approach for managing deviations from realistic targets under specified priorities.

The work proceeds in six phases. In the first phase, the feed mixture problem is analyzed and the relevant criteria are identified. In the second phase, a model is developed that combines the various criteria in order to find the best possible ingredient mixtures. The third phase assigns the particular input data used to solve the feed mixture problem for fattening pigs. Phases four and five are the creation of the multicriteria and goal programming models that solve the identified problem and the presentation of four scenarios that exploit the decision maker's preferences, respectively. The sixth phase analyzes the results of the previous phases. The remainder of this paper is organized accordingly: Section 2 presents the multicriteria model, the input data, and the goal programming formulation; Section 3 reports the marginal solutions, the payoff analysis, the scenario results, and the sensitivity analysis; Section 4 discusses the findings in the context of the literature; and Section 5 concludes.

2 Materials and Methods

2.1Model for multicriteria programming

When formulating the model to determine the composition of a feed mixture, several factors must be taken into account:

- the costs associated with preparing the mixture;

- the specific dietary requirements of the animals for which the mixture is intended;
- the overall quality of the mixture being produced [27].

As a result, the optimal feed mixture would ideally minimize preparation costs, fully satisfy the nutritional requirements of the animals, and maximize the quality of the feed. Consequently, the formulation of an optimal feed mixture will necessitate consideration of the following criteria:

- cost, measured in monetary units;
- nutrient composition (expressed as a percentage) necessary for achieving maximal weight gain in animals;
- nutrient composition (expressed as a percentage) that influences the overall quality of the mixture, thereby affecting the weight gain of the animals for which the feed is prepared.

The second criterion aims to develop a blend of components that maximizes weight gain. Weight gain depends on obtaining as many nutrients as possible, with a focus on nutrient elements that are highly digestible. Greater digestibility results in increased weight gain per unit of feed blend, which in turn decreases the actual feed cost to the producer over the longer term [28].

The purpose of the third criterion is to improve the quality of the mixture, specifically with regard to shelf life, by ensuring that a minimum amount of water (moisture) is present. Low moisture content makes the same weight gain possible with less water and therefore less feed, thereby reducing feed costs [29].

All three criteria are interconnected and conflict with one another. In particular, the most expensive feeds are often the ones that provide high levels of ingredients that are good for weight gain and tend to be low in moisture; but a nutrient-rich feed for weight gain is not always a low-moisture feed [30]. This difference is reflected in the data inputs for the three criteria in Table 1.

Since the objectives of minimizing feed cost, maximizing nutrient value, and minimizing moisture content are conflicting in nature, a single-objective optimization approach may not provide a satisfactory solution. Therefore, the feed formulation problem is modeled as a multicriteria programming problem, where the decision variables represent the proportions of individual feed ingredients included in the final mixture. The objective is to identify a compromise solution that simultaneously satisfies the nutritional requirements of the animals while achieving an appropriate balance among cost, nutrient content, and feed quality.

Table 1. Cost, nutrient content, and moisture composition of feed ingredients used in the feed formulation optimization

Variable	**Feed**	**Cost (C_{i1}), min**	**Nutrients (C_{i2}), max**	**Moisture (C_{i3}), min**
x_1	Barley	28	70	12.7
x_2	Maize	21	80	13.7
x_3	Wheat	15	79	13.3
x_4	Palm kernel cake, expeller	12	51	8.1
x_5	Molasses, sugar beet	20	86	21.3
x_6	Soybean meal, solvent extracted	20	83	11.8
x_7	Fish meal, treated	35	69	8.9
x_8	Bone meal	35	68	5.2
x_9	Groundnut expeller, dehulled	45	79	6.8
x_{10}	Maize bran	15	63	10.6
x_{11}	Wheat bran	28	69	13.1
x_{12}	Brewers' grain, dehydrated	70	70	8.5

Source: adapted from published feed formulation optimization literature; cost, nutrient quality index, and moisture content values were used as input parameters in the multicriteria programming model. C_{i1} = cost coefficient (minimization objective); C_{i2} = nutrient content coefficient (maximization objective); C_{i3} = moisture content coefficient (minimization objective).

The values reported under the nutrient criterion (C_{i2}) represent a nutritional quality index associated with the digestibility and weight-gain potential of each feed ingredient. Higher values indicate a greater contribution toward animal growth performance and feed utilization efficiency; therefore, the objective is to maximize the overall nutritional value of the feed mixture.

All three criteria have an economic impact: they all affect business performance and thus profits to some extent. Cost reduction has direct implications for the business, and the quality of the feed blend has indirect impacts through the prevention of deficiencies in production quality, improved weight gain of the animals, and the quality of the end product (meat) [31]. Improved product quality contributes to improved prices, which in turn contribute to overall business performance. It is essential to note that minimizing feed costs does not guarantee the best business outcome, and maximizing the quality of the feed mixture does not necessarily ensure optimal performance. For the company to perform best, each blend must be optimally balanced between cost and quality. In order to determine the optimal acceptable levels of the criterion functions, the relationship between the criterion functions and overall performance must be established, and this should involve the decision maker [32].

The problem of finding the optimal feed mixture is indeed a multicriteria decision-making (MCDM) problem, as pointed out by Annetts and Audsley [33]. To solve it with MCDM methods, a multicriteria linear programming model is first formulated. This involves defining a set of objective functions and constraints that include the different criteria—minimizing cost, maximizing weight gain, and optimizing feed quality. The aim is to identify the composition of the feed mixture that fulfils the criteria while meeting the nutritional requirements of the animals and the economic objectives of the company [34]. The multicriteria model can be expressed mathematically as follows:

$$\text{Min (Max) } Z_p = \sum_{i=1}^{n} C_{ip}\, x_i, \qquad (p = 1,2,3) \qquad (1)$$

$$\sum_{i=1}^{n} a_{ij}\, x_i \leq (\geq)\, b_{jq}, \qquad (j = 1, \dots, m) \qquad (2)$$

$$\sum_{i=1}^{n} x_i = 1 \qquad (3)$$

$$x_i \geq 0, \qquad (i = 1, \dots, n) \qquad (4)$$

where Z_p $(p = 1,2,3)$ is the optimization function for criterion p; m is the number of different nutrient requirements for a particular kind and category of animal; b_{jq} is the requirement for nutrient j for animal category q; n is the number of available feeds; C_{ip} is the coefficient of feed i in criterion function p; x_i is the quantity of feed i in the mixture; and a_{ij} is the quantity of nutrient j per unit of feed i.

In addition to the essential nutritional requirements (represented by b_j), there may be other limitations to consider, such as the maximum allowable intake of certain nutrients. It is common for there to be restrictions on the quantity of specific types of feed, ensuring they are neither excessively abundant nor insufficient. These constraints vary depending on the animal species and the recommendations provided by nutritionists.

Equation (3) is established during the formulation of the mixture recipe; its left-hand side may be set to less than 1 to allow flexibility, regardless of the composition of the mixture ingredients or additives.

The first step is to create a multicriteria linear programming model that is used to determine the optimal feed mixture. This model is then converted into a goal programming model based on nutritionists' recommendations [35]. What distinguishes this work from similar studies is the inclusion of two new criteria that measure the quality of the feed mix: the digestibility and the moisture content of the optimum mixture. In addition, we show how decision makers can use the model [36] to interactively modify their preferences for the stated objectives or to understand how best to make decisions.

Because the three objectives are conflicting in nature, it is generally impossible to achieve their individual optimum values simultaneously. Consequently, the multicriteria linear programming model serves as the foundation for the development of a goal programming model, where target values are assigned to each objective and deviations from these targets are minimized according to the priorities specified by the decision maker.

2.2Input data for determining the feed mixture for three stages of pig growth

Data play an important role in creating the optimum feed formula for three types of pig feed: P1 (pig starter), P2 (pig grower), and P3 (pig finisher). P1 is the formulation appropriate for pigs weighing up to 20 kg, P2 for pigs weighing 20–50 kg, and P3 for pigs weighing 50–100 kg. Each blend should contain both the minimum and maximum amounts of each nutrient per day. These nutrient ranges are based on scientific research and on the data shown in Tables 1–3.

When developing a feed mixture for this livestock category, the ingredients utilized, their unit prices, and the amount of nutrients and moisture per unit of each ingredient are shown in Table 1. The main goals are to keep the total cost as low as possible, to make as large a percentage of the mix as possible consist of nutrients, and to achieve the smallest possible percentage of moisture in the optimal meal.

Table 2 shows the minimum necessary nutrient levels in the feed for the three categories of pigs, with the prescribed requirements—minimum CP (%), minimum crude fat (%), maximum crude fibre (%), maximum total ash (%), and maximum moisture content (%)—taken from the Bureau of Indian Standards (BIS). Estimates of Ca (%) and P (%) are taken from NRC (2012) [37]. Estimates of ME for pigs vary from 92 to 160 kcal/kg $BW^{0.75}$, with an average estimate of 106 kcal/kg $BW^{0.75}$ (NRC, 1998). According to NRC (2012) [37], the estimated requirement of ME for growing and finishing pigs is 197 kcal per 600 lb of body weight. Moisture content is not restricted differently across the three categories; it is set to 11% of the total mixture, meaning that the dry matter of the mixture should be 89%.

Table 2. Daily nutrient requirements for the three growth phases

	Nutrient	P1 (b_{j1})	P2 (b_{j2})	P3 (b_{j3})
N1	Moisture content (max %)	11	11	11
N2	Crude protein (min %)	20	18	16
N3	Crude fat or ether extract (min %)	2	2	2
N4	Crude fibre (max %)	5	6	8
N5	Total ash (max %)	8	8	8
N6	Metabolizable energy (kcal/kg), min	1513	2834	2900

Some nutrients are required in small amounts (Table 2) and others in large amounts. Table 3 is the nutrition matrix, each element a_{ij} of which is the concentration of a particular nutrient per unit of feed. Data for all feed ingredients were taken from *Nutrient Requirements and Feed Ingredient Composition for Pigs*, CVB-series No. 64 (October 2020), and from NRC (1998).

Table 3. Nutritional composition and metabolizable energy values of the feed ingredients used in the multi-objective feed formulation model

Variable	Feed	DM (%) a_{i1}	CP (%) a_{i2}	Crude fat (%) a_{i3}	Crude fibre (%) a_{i4}	Ash (%) a_{i5}	Ca (%) a_{i6}	P (%) a_{i7}	ME (kcal/kg) a_{i8}
x_1	Barley	87.3	10.2	2.7	4.7	2.1	0.05	0.32	3073.0
x_2	Maize	86.3	7.5	3.8	2.4	1.2	0	0.25	3395.0
x_3	Wheat	86.7	11	2.1	2.3	1.5	0.03	0.29	3376.0
x_4	Palm kernel cake, expeller, CF < 180 g/kg	91.9	15.9	8.7	16.7	4.2	0.29	0.59	3063.0
x_5	Molasses, sugar beet	78.7	9.8	0.2	–	9	0.07	0.05	2298.0
x_6	Soybean meal, solvent extracted, HiPro, CF < 45 g/kg, CP < 480 g/kg	88.2	46.9	2.4	3.7	6.5	0.29	0.67	3369.0
x_7	Fish meal, treated, CP < 600 g/kg	91.1	56.1	14.2	0.22	19.4	4.01	2.64	3528.0
x_8	Bone meal	94.8	40.2	5.1	–	46.3	17.77	8.23	1961.0
x_9	Groundnut expeller, dehulled, CF < 75 g/kg	93.2	47.6	9.4	6.1	6.4	0.23	0.48	3109.0
x_{10}	Maize bran	89.4	9.3	4.8	9.1	2.3	0.03	0.47	2584.0
x_{11}	Wheat bran, CF < 125 g/kg	86.9	14.9	4.4	10.9	5.3	0.1	1.06	2318.0
x_{12}	Brewers' grain, dehydrated	91.5	24.8	7	13.2	4.6	0.35	0.46	1920.0

DM = dry matter; CP = crude protein; CF = crude fibre; ME = metabolizable energy. Source: CVB-series No. 64 (2020) and NRC (1998).

2.3Multicriteria linear model (Model 1)

Based on the data in the tables above and taking into consideration the preferences of the decision makers, a multicriteria linear programming model is developed for all three types of mixture. The model reduces the cost of the mixture and its moisture content while increasing the total nutrient value [38].

Tables 1, 2, and 3 provide the objective-function coefficients and the nutritional and composition constraints for P1–P3, respectively, and the following multicriteria model can be specified. The model comprises three objective functions, 12 decision variables ($x_1, x_2, \dots, x_{12}$), and six nutritional constraints from Tables 2 and 3. There are three distinct problems with identical objective functions (5), (6), and (7) and constraints (9) and (10), while constraint (8) varies for each of the three categories.

$$\text{Min } Z_1 = \sum_{i=1}^{n} C_{i1}\, x_i \qquad \text{(cost)} \qquad (5)$$

$$\text{Max } Z_2 = \sum_{i=1}^{n} C_{i2}\, x_i \qquad \text{(nutrients)} \qquad (6)$$

$$\text{Min } Z_3 = \sum_{i=1}^{n} C_{i3}\, x_i \qquad \text{(moisture)} \qquad (7)$$

$$\sum_{i=1}^{n} a_{ij}\, x_i \le (\ge)\, b_{jq}, \qquad (j = 1, \dots, m;\ q = 1,2,3) \qquad (8)$$

$$\sum_{i=1}^{n} x_i = 0.9 \qquad (9)$$

$$0 \le x_i \le 0.15, \qquad (i = 1, \dots, 12) \qquad (10)$$

Constraint (9) represents relationship (3) of the initial model with a right-hand-side value of 0.9. Specifically, the recommended diet consists of a 1 kg mixture to which 1–3% pig premix and 0.3–0.5% salt are added according to NRC (2012) [37]; these additives are disregarded among the ingredients included in the optimal mixture.

The model also incorporates further limits on the quantity of specific feeds. In order to create a diverse diet, the model restricts the proportion of any single feed to a maximum of 15%. Therefore, if x_i represents the proportion of feed i in the ideal mixture, the constraints $x_i \le 0.15$ $(i = 1, \dots, 12)$ are imposed. The final two relationships in Model 1 exhibit this structure.

The base solutions of the three problems for each objective function, called the marginal solutions, are computed separately.

2.4Goal programming model (Model 2)

Upon reaching a consensus with the nutritionist, it was decided to resolve all three problems by transforming the multicriteria model into a goal programming model. The goal values for the three objective functions were aligned with the marginal solutions obtained from Model 1 (Section 3.1). The goals were as follows.

For P_1: to obtain a diet plan with a cost of 16.92 monetary units ($Z_{11} = 16.92$); a nutritional composition of 71.4% in the feed mixture ($Z_{21} = 71.4$); and a moisture share of 8.67% ($Z_{31} = 8.67$).

For P_2: to obtain a diet plan with a cost of 17.09 monetary units ($Z_{12} = 17.09$); a nutritional composition of 71.4% ($Z_{22} = 71.4$); and a moisture share of 8.71% ($Z_{32} = 8.71$).

For P_3: to obtain a diet plan with a cost of 17.77 monetary units ($Z_{13} = 17.77$); a nutritional composition of 70.6% ($Z_{23} = 70.6$); and a moisture share of 8.95% ($Z_{33} = 8.95$).

The goal programming model is formulated by reformulating the three objective functions as goal constraints. Realizing all three simultaneously is difficult, if not impossible. Deviation variables are therefore added to the three goal functions to convert them into constraints, and the objective function of the model is the sum of the deviation variables, which is minimized. This implies that, for any goal, the solution may be larger or smaller than the goal value. The goal programming model for each of the three categories (P_1–P_3) is given below, where U_p and O_p denote the under- and over-achievement of goal p, respectively.

$$\text{Min } Z = \sum_{p=1}^{3} \left(U_p + O_p\right) \qquad (11)$$

subject to

$$\sum_{i=1}^{12} C_{i1}\, x_i + U_1 - O_1 = Z_{1n} \qquad (12)$$

$$\sum_{i=1}^{12} C_{i2}\, x_i + U_2 - O_2 = Z_{2n} \qquad (13)$$

$$\sum_{i=1}^{12} C_{i3}\, x_i + U_3 - O_3 = Z_{3n}, \qquad (n = 1,2,3) \qquad (14)$$

$$\sum_{i=1}^{12} a_{ij}\, x_i \ge (\le)\, b_{jq} \qquad (15)$$

$$\sum_{i=1}^{12} x_i = 0.90 \qquad (16)$$

$$0 \le x_i \le 0.15, \qquad (i = 1, \dots, 12) \qquad (17)$$

2.4.1 Scenarios for determining the constituents of the feed mixture

Three scenarios assign different lexicographic priorities ($P_1 > P_2 > P_3$) to exceeding the cost target, falling short of the nutrient target, and exceeding the moisture target, yielding distinct forms of the achievement function.

Scenario I: minimize cost overachievement first, then nutrient shortfall, then moisture excess.

$$Z = \text{Min}[P_1 O_1 + P_2 U_2 + P_3 O_3] \qquad (18)$$

Scenario II: minimize nutrient shortfall first, then cost overachievement, then moisture excess.

$$Z = \text{Min}[P_1U_2 + P_2O_1 + P_3O_3] \qquad (19)$$

Scenario III: minimize moisture excess first, then cost overachievement, then nutrient shortfall.

$$Z = \text{Min}[P_1O_3 + P_2O_1 + P_3U_2] \qquad (20)$$

A fourth scenario, which adds deviation variables to the crude protein constraint, is motivated by the results of Scenarios I–III and is introduced in Section 3.3. All models were solved in Google Colab, where a weight-based sensitivity analysis was also carried out (Section 3.4).

3 Results

3.1Marginal solutions and payoff analysis

The best solutions for each objective function of Model 1 are presented in Table 4. The values of the three objective functions at each of the marginal solutions are shown in the payoff table (Table 5).

Table 4. Optimal feed ingredient proportions corresponding to the individual objective functions (marginal solutions)

Feed	P1 Cost (min) S1	P1 Nutrient (max) S2	P1 Moisture (min) S3	P2 Cost (min) S1	P2 Nutrient (max) S2	P2 Moisture (min) S3	P3 Cost (min) S1	P3 Nutrient (max) S2	P3 Moisture (min) S3
x_1	0	0	0	0	0.03441	0	0.00562	0.05	0
x_2	0.04432	0.15	0	0.15	0.15	0.00922	0.15	0.15	0.09219
x_3	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
x_4	0.15	0	0	0.15	0	0.15	0.15	0	0.15
x_5	0.15	0.15	0	0.04022	0.15	0	0	0.1	0
x_6	0.15	0.15	0.1061	0.15	0.15	0.15	0.15	0.15	0.15
x_7	0.09335	0	0.15	0.10099	0.10819	0.15	0.15	0.15	0.15
x_8	0.01233	0.03475	0.04726	0.00879	0.0074	0.04466	0	0	0.04674
x_9	0	0.15	0.15	0	0.15	0.15	0	0.15	0.15
x_{10}	0.15	0	0.14664	0.15	0	0.09612	0.14438	0	0.01107
x_{11}	0	0	0	0	0	0	0	0	0
x_{12}	0	0.11525	0.15	0	0	0	0	0	0

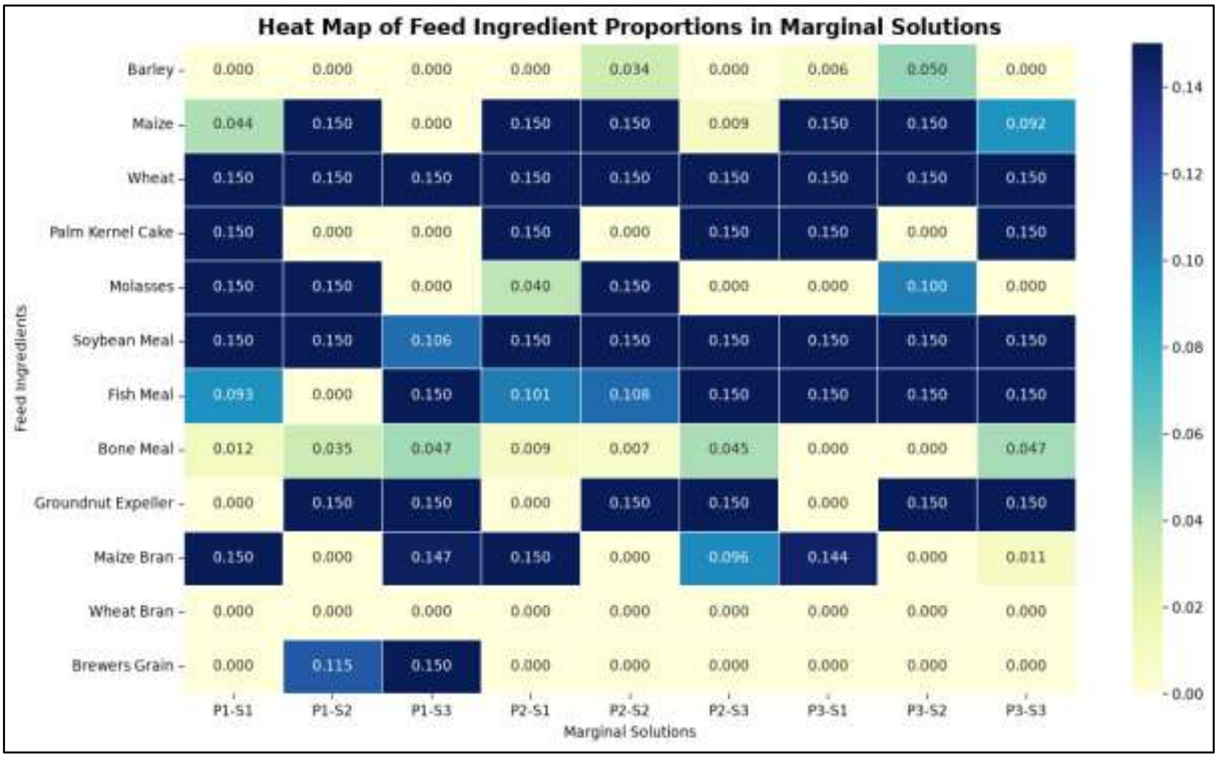

Figure 1. Heat map of feed ingredient proportions in the marginal solutions to the cost minimization (S1), nutrient maximization (S2), and moisture minimization (S3) problems for P1, P2, and P3. Darker colours indicate a higher proportion of the ingredient in the optimal formulation.

The marginal solutions are shown in Figure 1 as the different ingredients included in each solution. In most cases, the solutions have a high content of wheat and soybean meal, suggesting that these play a key role in providing nutrients. Other ingredients, such as bone meal, wheat bran, and brewers' grain, are used in some solutions but not in all. The observed differences illustrate how the optimization model minimizes feed cost while simultaneously achieving the required nutrient levels and reducing moisture content.

Table 5. Payoff matrix for the multi-objective feed formulation problem

Objective	P1-S1	P1-S2	P1-S3	P2-S1	P2-S2	P2-S3	P3-S1	P3-S2	P3-S3
Cost, Z_1	16.9295	27.4338	30.7257	17.0967	23.1591	22.2485	17.7731	23.8	22.7879
Nutrients, Z_2	65.1252	71.4805	65.8083	64.425	71.427	63.98	63.7894	70.6	65.4009
Moisture, Z_3	11.2671	11.1953	8.67713	10.4262	11.4734	8.7124	9.97181	10.94	8.95841

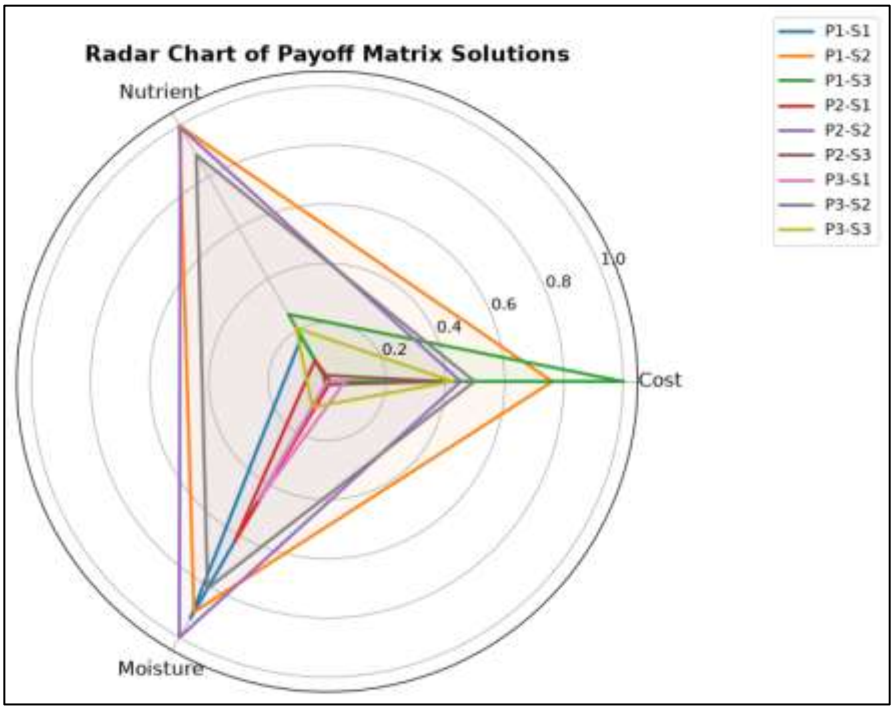

Figure 2. Radar chart of the payoff matrix solutions (normalized trade-off analysis).

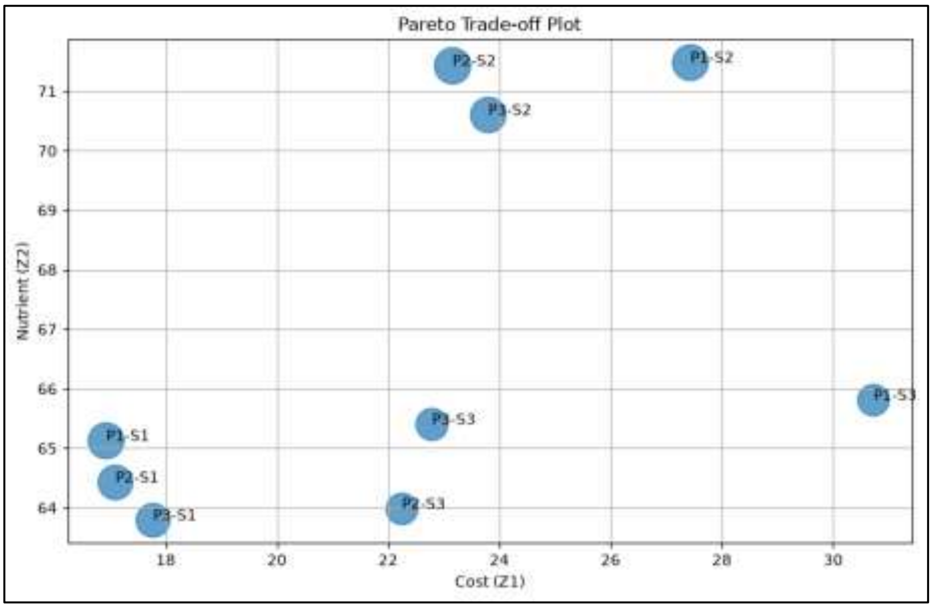

Figure 3. Pareto trade-off plot of cost against nutrient value, with moisture content indicated for each marginal solution.

The payoff table (Table 5) summarizes the results of the three objective functions when each is optimized individually. The diagonal entries contain the optimal values for cost minimization (Z_1), nutrient maximization (Z_2), and moisture minimization (Z_3), while the off-diagonal entries show the performance of the other two objectives at each marginal solution. These results show clearly that the objectives conflict: no single marginal solution is capable of minimizing moisture and cost without compromising the nutrient level.

For P1, the minimum-cost solution (S1) achieves a cost of 16.93. However, when nutrient maximization (S2) is prioritized, the cost increases to 27.43, an increase of approximately 62%. Similarly, the moisture-minimization solution (S3) results in a cost of 30.73, approximately 81% higher than the minimum-cost solution. Similar trade-off patterns are observed for P2 and P3, confirming that improvements in one objective are generally accompanied by deterioration in one or more of the remaining objectives.

These trade-offs are further illustrated in Figure 2, a radar chart showing a normalized trade-off analysis of the payoff solutions. The figure emphasizes the differences in objective performance among the solutions, and it is evident that no solution is superior to all others on all three criteria. Similarly, Figure 3 shows the Pareto relationship between cost and nutrient value, taking moisture levels into consideration. The distribution of solutions illustrates that more expensive solutions generally contain higher nutrient values and less moisture, and that other combinations can be obtained with different ingredients and/or objective priorities.

Figures 2–3 and Table 5 show that pursuing one objective affects the others. Hence, a compromise decision-making process capable of weighing and balancing the objectives is required. To this end, the problem is reformulated using goal programming (Model 2), in which the marginal optima are set as target values for each objective and the positive and negative deviations from these targets are controlled separately. This allows the development of balanced feed rations that take into account economic optimization, nutrient intake, and moisture content.

3.2Goal programming scenarios

The optimal formulations and objective function values obtained under Scenarios I–III for each growth phase are reported in Table 6 and visualized in Figure 4.

Table 6. Optimal feed formulations and objective function values obtained under the three lexicographic scenarios

Feed	P1 Sc. I	P1 Sc. II	P1 Sc. III	P2 Sc. I	P2 Sc. II	P2 Sc. III	P3 Sc. I	P3 Sc. II	P3 Sc. III
x_1	0.000	0.078	0.000	0.000	0.000	0.000	0.093	0.000	0.000
x_2	0.054	0.150	0.028	0.150	0.150	0.007	0.150	0.150	0.117
x_3	0.150	0.150	0.150	0.150	0.150	0.150	0.150	0.150	0.150
x_4	0.150	0.000	0.000	0.150	0.000	0.150	0.150	0.134	0.150
x_5	0.150	0.150	0.000	0.066	0.150	0.000	0.000	0.068	0.000
x_6	0.150	0.150	0.150	0.150	0.150	0.150	0.150	0.150	0.150
x_7	0.101	0.000	0.141	0.103	0.127	0.131	0.105	0.098	0.098
x_8	0.002	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000
x_9	0.000	0.150	0.150	0.000	0.150	0.150	0.000	0.150	0.150
x_{10}	0.144	0.007	0.150	0.131	0.000	0.150	0.102	0.000	0.085
x_{11}	0.000	0.031	0.000	0.000	0.023	0.000	0.000	0.000	0.000
x_{12}	0.000	0.000	0.130	0.000	0.000	0.012	0.000	0.000	0.000
Total	0.900	0.900	0.900	0.900	0.900	0.900	0.900	0.900	0.900
Cost	**16.920**	**22.491**	**28.921**	**17.090**	**23.242**	**21.626**	**18.003**	**21.552**	**20.956**
Nutrients	**65.276**	**71.400**	**66.740**	**64.986**	**71.400**	**63.688**	**64.128**	**67.594**	**65.279**
Moisture	**11.342**	**11.684**	**9.128**	**10.746**	**11.465**	**8.953**	**10.231**	**10.245**	**9.378**

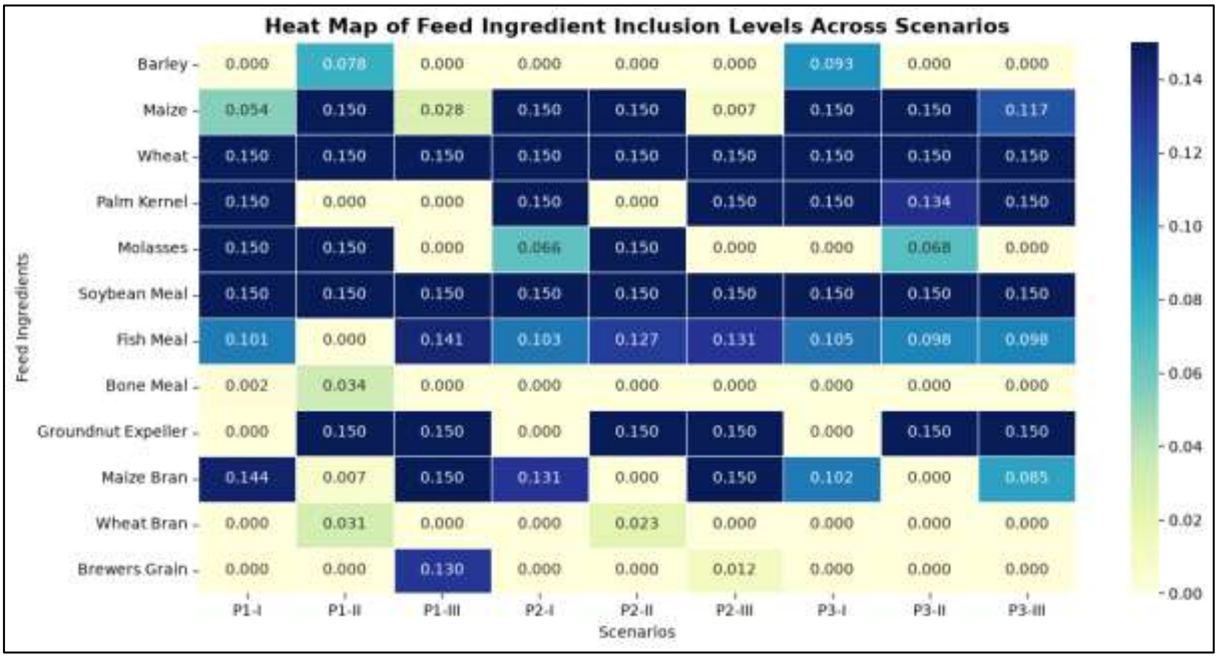

Figure 4. Heat map of feed ingredient inclusion levels across the nine scenario solutions.

The changes in feed ingredient inclusion rates across the nine optimized formulations produced by the multi-objective model are shown in Figure 4. The colour intensity represents the proportion of each feed in the optimal mixture: darker colours indicate higher proportions and lighter colours indicate lower proportions or the absence of a particular feed.

For some ingredients there is an obvious trend. Across all scenarios, wheat (x_3) and soybean meal (x_6) were included at their maximum levels, which indicates that both are important for the nutrition of the formulated feeds. Their consistent presence reflects their contribution to nutrient adequacy while retaining practical compatibility in formulation.

Notable variations exist in the inclusion of several other ingredients, such as palm kernel cake (x_4), molasses (x_5), groundnut expeller (x_9), and maize bran (x_{10}), across the scenarios. These changes arise from the way in which the optimization model adjusts their proportions according to the priority given to minimizing cost, maximizing nutrient content, or reducing moisture content.

Most scenarios include moderate amounts of fish meal (x_7), which is rich in protein. However, bone meal (x_8), wheat bran (x_{11}), and brewers' grain (x_{12}) occur only in certain solutions, and thus become significant only under certain combinations of objectives and constraints.

Overall, the heat map helps visualize how the feed composition changes according to the optimization objectives. The outcome shows that the model is flexible enough to generate alternative feed formulations that satisfy given economic, nutritional, and moisture criteria.

In Scenario I, the main criterion was low feed expense. The results indicate that this approach lowered the cost to 16.920 units for P1, which is very close to the desired cost targets for P1, P2, and P3. The cost reduction is, however, offset by higher moisture content and a lower nutrient level, as reported in [39,40], where nutrient imbalance in feeding strategies has been observed and cost saving may affect animal performance.

In contrast, Scenario II minimized the nutrient shortfall. The rise in cost (22.491 units for P1) is attributable to the higher level of nutrient-rich ingredients needed to meet biological requirements, as previously observed by Makkar [41], who stated that rations with greater nutrient content, formulated to satisfy the biochemical needs of swine, tend to be more costly and require more investment and resources in general. There was also an increase in moisture content as a result of the higher moisture of the nutrient-rich ingredients selected.

Scenario III, which minimized moisture excess, yielded the lowest moisture content (9.128 units for P1) and the highest cost (28.921 units). This is consistent with assessments of water use in livestock production systems [42], which indicate that moisture-efficient feeding strategies require more specialized and more expensive ingredients and make ration balancing more complex. Nutrient content in this scenario (66.740) lies between Scenarios I and II, which suggests a moderate balance between quality and nutrient content.

These findings are in line with the recent literature. Villalba et al. [43] demonstrated that multi-objective formulations that account for both economic and environmental priorities outperform single-objective cost minimization. Untea et al. [44] highlighted the importance of tailored feeding strategies that consider not only cost but also animal health and product quality. Monteiro and Gökdal [45] showed that optimizing feeds with water-efficiency constraints, as in Scenario III, supports sustainable production but must be carefully balanced against rising input costs; the trade-offs observed here echo industry-wide challenges in sustainable feed development. Moreover, the findings reinforce the insights of Akintan et al. [46] and Leen et al. [47], who advocated flexible constraint handling and dynamic goal prioritization to adapt feed composition to changing resource availability and market conditions.

3.3Controlling crude protein: Scenario IV

Across all three solutions for P1, P2, and P3, the quantity of crude protein significantly exceeds the specified minima: 21.10–28.60 units, 25.90–27.30 units, and 19.97–25.35 units, respectively, against minimum requirements of 20, 18, and 16 units. To adhere to standard requirements, a solution in which the protein quantity is not so large is desirable. This challenge parallels the overfeeding issues described by van Milgen and Dourmad [48], who highlighted the importance of protein precision to reduce both feed costs and the environmental footprint of swine production. Consequently, deviation variables U_4 and O_4 are introduced into the crude protein constraint, and O_4 is integrated into the achievement function to minimize the extent of protein oversupply. The introduction of deviation variables to control crude protein, coupled with adjustments to the achievement function, offers a practical pathway for producers to align feed formulation with sustainability goals and regulatory requirements for nutrient management, echoing the precision nutrition frameworks emphasized by Pomar and Remus [49]. The crude protein requirement is therefore reformulated as an equality with $b_1 = 20, b_2 = 18$, and $b_3 = 16$ for P1, P2, and P3, respectively, allowing for positive or negative deviations. The fourth scenario is defined as follows.

Scenario IV (applied to P1): the first priority minimizes cost overachievement, the second minimizes moisture excess and nutrient shortfall, the third penalizes protein oversupply, and the fourth penalizes protein undersupply, via O_4 and U_4 in the protein goal constraint:

$$Z = \text{Min}[P_1O_1 + P_2(O_3 + U_2) + P_3O_4 + P_4U_4] \qquad (21)$$

with $P_1 > P_2 > P_3 > P_4$, where the protein requirement is set as an equality with deviations to limit oversupply while preserving feasibility.

3.4Sensitivity analysis

The multi-objective model and the sensitivity analysis were implemented in Google Colab. The purpose of the sensitivity analysis is to examine the linear programming problem designed to optimize the feed mix and to determine how the optimal solution changes as the weights assigned to the objectives vary. The objective is to minimize a weighted combination of total cost, total moisture, and the negative of total nutrients (so that nutrients are maximized). The analysis investigates the optimal proportions (x values) of the ingredients when the weights given to cost, nutrients, moisture, and protein are changed, and how the resulting total cost, nutrients, and moisture change with the different weight combinations. For each scenario, the weight of each objective is varied to determine the corresponding trade-offs and how the weights influence the solution (Supplementary Table S1).

The best solutions identified by the proposed multi-objective optimization scheme under different priority scenarios for cost, nutrient content, moisture content, and protein content are given in Table S1. The four objectives receive different weights in each scenario so as to represent different practical decision contexts. The 12 decision variables $(x_1, x_2, \dots, x_{12})$ are the proportions obtained from the trade-offs between the competing goals. The resulting total cost, nutrient, moisture, and protein values illustrate the ability of the model to balance allocations against the desired weights. For instance, when cost minimization is dominant (weight = 100), the total cost is lowered to 16.92, but at the same time there is a significant gap in nutrient content (65.28 as opposed to the target of 71.4), as seen in the mean deviations (columns U_1–U_4). The moisture content lies in the range 9.23–12.03 and the protein content varies marginally (0.2–0.2738) across all scenarios, highlighting the flexibility of the model to reallocate proportions when priorities change.

The underachievement (U_1–U_4) and overachievement (O_1–O_4) values give a quantitative indicator of each goal's deviation from its target and are expressed explicitly as mean deviations within each criterion. The deviations, aggregated with the assigned weights, give the final objective value of the solution for each scenario, which ranges from 86.98 to 376.89 and is an index of the overall performance of the solution. The higher mean deviations in some objectives when other objectives are weighted heavily confirm the trade-offs inherent in multi-objective formulation problems. Notably, all scenarios reach an optimal state, which indicates that, within the bounds considered, the model finds feasible and balanced solutions. This listing of weighted trade-offs and mean deviations demonstrates that the proposed method is effective in supporting informed and adaptive choices for the efficient design of rations focused on meeting nutritional needs.

In summary, the multi-objective model developed in this paper is flexible and adaptive in practice, depending on the differing and sometimes conflicting needs of decision makers in defining the model or allocating resources. The basic model includes fundamental hard limits (Table 2) but can be modified to include new user-defined conditions. In this study, four illustrative scenarios were used, with different priority settings for the cost, nutrient, moisture, and protein goals. Although there are limits to the number of decision contexts that can be compressed into one model run, the framework is flexible enough to allow constraints to be added or modified to fit the needs of the decision maker. The decision maker needs to consider the conditions of the problem carefully, since overly demanding requirements may render the problem infeasible, which encourages rethinking or moderating some of the requirements to obtain a realistic and balanced solution. The result is a product that is as close to the practical objective as possible, with a clear view of the compromises that were made.

4 Discussion

In all scenarios, the balanced goal programming solutions are consistent with previous studies demonstrating that multi-objective diet formulation can bring about improvements (in environmental performance or nutrient sufficiency) while controlling increases in cost [50,51]. Lexicographic prioritization of the deviations around explicit targets as a method of handling competing targets in goal programming—including least cost, nutrient sufficiency, and the control of other handling or stability deviations such as moisture—is similar to that encountered in classical agricultural applications of goal programming [52–54]. The explicit moisture target for shelf life and feed handling was directionally consistent with feed mix studies in which water content is either restricted or penalized, with ingredients shifting in a predictable manner to satisfy the higher-priority targets with the lower deviations [55,56].

The nutrient-maximizing scenarios make sense in terms of the nutrient and energy system requirements and are consistent with phase-specific nutrient requirements in swine nutrition [37,57] and with the net energy concept, which explains the trade-offs and interactions among energy density, amino acid profiles, and cost [58–60]. The multi-objective behaviour observed here is also supported by recent optimization advances that involve linear and Bayesian multi-objective formulations for swine and for general feed design, using efficient frontiers formed by the weights or priorities on the various nutrient, cost, and other constraints [60,61].

The lexicographic goal programming solutions showed that cost growth was one of the least challenging objectives to satisfy, while other requirements such as acceptable feeding quality and animal performance could be met with high priority—similar to evidence that more than one objective can be addressed in multi-objective feeding optimization under targeted output growth and controlled economic trade-offs across livestock systems. The model's explicit inclusion of a moisture objective, in addition to nutrient sufficiency and cost, represents a real-world constraint on the ingredients in the mix that has been incorporated in other agricultural optimization studies and shifts the ingredients in predictable and policy-relevant ways. The findings highlight that the insights provided by the payoff table and the lexicographic priorities are not merely a methodological convenience but a useful decision lever to guide ration design for optimal alignment with real-world objectives over different growth phases and types of procurement.

Several simplifications and possible extensions should be noted, as they motivate further research: sensitivity analysis of the target levels, NE-based constraints, and optimization under ingredient-composition uncertainty. Future research can include the explicit integration of environmental goals and can make use of recent developments in multi-objective Bayesian optimization to extract efficient frontiers and assess trade-offs in the face of market variations and noise in the data, as developed by the swine diet optimization and feed tooling community. Finally, deployment in routine feed mill operations and benchmarking against least-cost baselines would allow the compromise solutions to be validated as credible benefits in practice for performance, procurement, and sustainability.

5 Conclusions

Based on the results of this study, lexicographic goal programming can be considered an appropriate tool for obtaining a set of balanced pig feed formulations that satisfy high-priority pig specifications without ignoring the requirements for animal nutrition and handling or the growth in cost, and that overcome the limitations revealed by the payoff analysis of the scenarios considered. As seen in the result tables and in the varying proportions of ingredients across scenarios, the deviations from the realistic targets set in the model are predictable and relate to the nutrient sufficiency of the final product, its moisture content, and its cost. The approach remains dependent on target setting and priority weights and on the assumed composition of the ingredients; extensions to the net energy approach, the handling of uncertainty, and explicit environmental goals, as mentioned in the Discussion, are therefore needed, and regular sensitivity analysis is encouraged. The framework is readily transferable to other livestock systems and menu planning efforts and can be tested in feed mill processes to confirm procurement, performance, and sustainability benefits relative to least-cost baselines.

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Conflict of Interest

The authors declare no conflict of interest.

Data Availability Statement

The ingredient composition data used in this study are taken from the published sources cited in Tables 1–3. The Google Colab notebooks used for the goal programming models and the sensitivity analysis (Supplementary Table S1) are available from the corresponding author on reasonable request.

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